RODOLFO GUIDOBONO	CURSO DE MECÁNICA TEÓRICA J. GARCÍA - EJERCICIOS CAP. 42	HOJA N° 1 FECHA 28/11/18

EJERCICIO 1 Demostrar usando la definición del corchete de Poisson que

$$\{H, a_k^*\} = -i\omega\, a_k^*$$

$$H = \int_{-\infty}^{\infty} dx \left[\frac{\Pi_{(x)}^2}{2} + \frac{(\phi'_{(x)})^2}{2} + \frac{m^2}{2}\phi_{(x)}^2 \right]$$

$$a_k^* = \int_{-\infty}^{\infty} dx \left[-i\,\Pi_{(x)} + \omega\,\phi_{(x)} \right] e^{-i(\omega t - kx)}$$

$$\{H, a_k^*\} = \int_{-\infty}^{\infty} dx \left\{ \frac{\delta H}{\delta \phi_{(x)}} \frac{\delta a_k^*}{\delta \Pi_{(x)}} - \frac{\delta a_k^*}{\delta \phi_{(x)}} \frac{\delta H}{\delta \Pi_{(x)}} \right\}$$

(1) $\dfrac{\delta a_k^*}{\delta \Pi_{(x)}} = -i\, e^{-i(\omega t - kx)}$ (2) $\dfrac{\delta a_k^*}{\delta \phi_{(x)}} = \omega\, e^{-i(\omega t - kx)}$

(3) $\dfrac{\delta H}{\delta \Pi_{(x)}} = \Pi_{(x)}$

Recordando que $\dfrac{\delta F(\phi_{(y)})}{\delta \phi_{(x)}} = \dfrac{\delta F}{\delta \phi_{(x)}} - \dfrac{\partial}{\partial x}\dfrac{\partial F}{\partial(\phi'_{(x)})}$

$$\frac{\delta H}{\delta \phi_{(x)}} = -\phi''_{(x)} + m^2\phi_{(x)}$$

KLEIN - GORDON $\longrightarrow$ $\ddot{\phi}_{(x)} - \phi''_{(x)} + m^2\phi_{(x)} = 0$ entonces

$$\frac{\delta H}{\delta \phi_{(x)}} = -\ddot{\phi}_{(x)} \quad \text{con} \quad \phi_{(x)} = \frac{1}{2}\int_{-\infty}^{\infty} \frac{dk}{2\pi\omega}\left[a_k e^{-i(\omega t - kx)} + a_k^* e^{i(\omega t - kx)} \right]$$

$$\dot{\phi} = \frac{1}{2}\int_{-\infty}^{\infty} \frac{dk}{2\pi\omega}\left[a_k(-i\omega) e^{-i(\omega t - kx)} + a_k^*(i\omega) e^{i(\omega t - kx)} \right]$$

$$\ddot{\phi} = \frac{1}{2}\int_{-\infty}^{\infty} \frac{dk}{2\pi\omega}\left[a_k(-i\omega)^2 e^{-i(\omega t - kx)} + a_k^*(i\omega)^2 e^{i(\omega t - kx)} \right]$$

$$= (i\omega^2)\,\frac{1}{2}\int_{-\infty}^{\infty} \frac{dk}{2\pi\omega}\left[a_k e^{-i(\omega t - kx)} + a_k^* e^{i(\omega t - kx)} \right]$$

$$\ddot{\phi}_{(x)} = -\omega^2\phi_{(x)} \longrightarrow (4)\ \frac{\delta H}{\delta \phi_{(x)}} = \omega^2\phi_{(x)}$$

NOTA

introduciendo ① ② ③ y ④ en la definición del corchete de Poisson

$$\{H, a_k^*\} = \int_{-\infty}^{\infty} dx \left\{ \omega^2 \phi_{(x)} \left(-i\, e^{-i(\omega t - kx)}\right) - \omega\, e^{-i(\omega t - kx)}\, \Pi(x) \right\}$$

$$= -i\omega \int_{-\infty}^{\infty} dx \left\{ \omega \phi_{(x)} + \frac{1}{i} \Pi(x) \right\} e^{-i(\omega t - kx)}$$

$$= -i\omega \int_{-\infty}^{\infty} dx \left(-i\Pi(x) + \omega \phi_{(x)} \right) e^{-i(\omega t - kx)}$$

$$\boxed{\{H, a_k^*\} = -i\omega\, a_k^*} \qquad \text{Q.E.D.}$$

<u>EJERCICIO 2</u> Demostrar que $\{H, a_k\} = i\omega a_k$

Usando las propiedades del corchete de Poisson y teniendo que

$$H = \frac{1}{4} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \left(a_k a_k^* + a_k^* a_k \right)$$

$$\{H, a_k\} = \left\{ \left[\frac{1}{4} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \left(a_k a_k^* + a_k^* a_k \right) \right], a_k \right\}$$

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$$= \frac{1}{4} \int_{-\infty}^{\infty} \frac{dq}{2\pi} \left\{ (a_q a_q^* + a_q^* a_q), a_k \right\}$$

Recordando que $\{a_k, a_q\} = 0$

$$\{a_k, a_q^*\} = -i\, 4\pi\, \omega\, \delta(k-q)$$

$$\{a_q^*, a_k\} = i\, 4\pi\, \omega\, \delta(k-q)$$

$$\{(a_q a_q^* + a_q^* a_q), a_k\} = \{a_q a_q^*, a_k\} + \{a_q^* a_q, a_k\}$$

$$\{a_q a_q^*, a_k\} = a_q \{a_q^*, a_k\} + \{a_q, a_k\} a_q^* = a_q \{a_q^*, a_k\}$$

$$= a_q\, i\, 4\pi\, \omega\, \delta_{(k-q)}$$

$$\{a_q^* a_q, a_k\} = a_q^* \{a_q, a_k\} + \{a_q^*, a_k\} a_q = \{a_q^*, a_k\} a_q$$

$$= i\, 4\pi\, \omega\, \delta_{(k-q)}\, a_q$$

HOJA Nº 3

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$$\{H, a_k\} = \frac{1}{4}\int_{-\infty}^{\infty}\frac{dq}{2\pi}\left(a_q\, i\, 4\pi\omega\,\delta(k-q) + i\,4\pi\omega\,\delta(k-q)\, a_q\right)$$

$$= \frac{1}{4}\int_{-\infty}^{\infty}\frac{dq}{\cancel{2\pi}}\, i\, \overset{4}{\cancel{8\pi}}\,\omega\, a_q\, \underbrace{\delta(k-q)}_{k=q}$$

$$\boxed{\{H, a_k\} = i\,\omega\, a_k}$$

NOTA